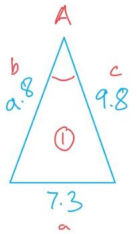
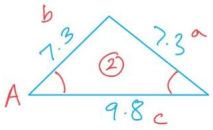
Q1

1

An isosceles triangle has side lengths 7.3 cm and 9.8 cm. Calculate the difference between the two possible smallest angles.

[4]





COSINE RULE

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

① $\cos A = \frac{2\times 9.8^2 - 7.3^2}{2\times 9.8^2}$

$A = \cos^{-1}(0.7225\ldots)$

$A = 43.733\ldots \quad A = 43.7°$

② $\cos A = \frac{7.3^2+9.8^2-7.3^2}{2(7.3)(9.8)}$

$A = \cos^{-1}(0.6712\ldots)$

$A = 47.837\ldots \quad A = 47.8°$

$47.8 - 43.7 = 4.1°$

4.1°

Q2a

2a

A triangle ABC has side lengths $AB = 3x$ cm, $BC = 5x$ cm and $AC = 6x$ cm.

(a) Calculate the size of the angle BAC to two decimal places.

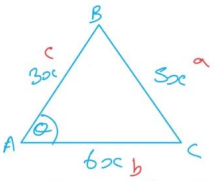
[2]

(b) Given that the total perimeter of the triangle is 37.8 cm, find the area of the triangle, correct to three significant figures.

[4]

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

a)



$\cos\theta = \frac{(6x)^2+(3x)^2-(5x)^2}{2(6x)(3x)}$

$\frac{36x^2+9x^2-25x^2}{36x^2} = \frac{20}{36}$

$\cos^{-1}\left(\frac{20}{36}\right) = 56.25101\ldots$

BAC = 56.25°

Q2b

2b

A triangle ABC has side lengths $AB = 3x$ cm, $BC = 5x$ cm and $AC = 6x$ cm.

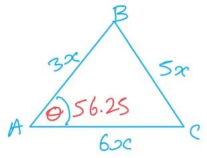
(a) Calculate the size of the angle BAC to two decimal places.

[2]

(b) Given that the total perimeter of the triangle is 37.8 cm, find the area of the triangle, correct to three significant figures.

[4]

b)



$3x + 5x + 6x = 37.8$

$14x = 37.8$

$x = 2.7$

$3x = 8.1$

$6x = 16.2$

$\text{AREA} = \frac{1}{2}bc\sin A$

$= \frac{1}{2}(8.1)(16.2)\sin 56.25$

$= 54.5527...$

$54.6\ \text{cm}^2$

Q3a

3a

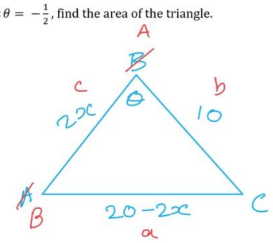
In a triangle ABC, $AB = 2x$ cm, $BC = 10$ cm and $AC = (20 - 2x)$ cm, angle $ABC = \theta°$.

(a) Show that $\cos\theta = \frac{4x - 15}{2x}$.

[2]

(b) Given that $\cos\theta = -\frac{1}{2}$, find the area of the triangle.

[4]



$\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$

a)

$\cos\theta = \frac{10^2 + (2x)^2 - (20-2x)^2}{2(10)(2x)}$

$\cos\theta = \frac{100 + 4x^2 - (400 - 80x + 4x^2)}{40x}$

$= \frac{100 + 4x^2 - 400 + 80x - 4x^2}{40x}$

$\frac{80x - 300}{40x}$ SIMPLIFY $\div 20$

$\cos\theta = \frac{4x - 15}{2x}$

Q3b

3b

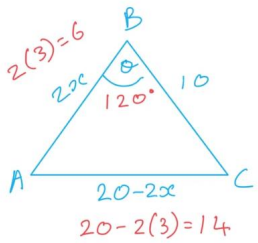
In a triangle ABC, $AB = 2x$ cm, $BC = 10$ cm and $AC = (20 - 2x)$ cm, angle $ABC = \theta°$.

(a) Show that $\cos\theta = \frac{4x-15}{2x}$.

[2]

(b) Given that $\cos\theta = -\frac{1}{2}$, find the area of the triangle.

[4]



b)

$\frac{4x-15}{2x} = -\frac{1}{2}$

$\cos\theta = -\frac{1}{2}$

$\cos^{-1}\left(-\frac{1}{2}\right)$

$\theta = 120°$

$4x - 15 = -x$

$5x = 15$

$x = 3$

$\text{AREA} = \frac{1}{2}ac\sin B$

$\frac{1}{2}(10)(6)\sin 120$

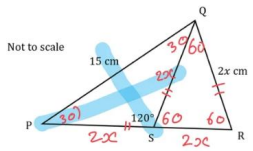
$= 25.980\ldots$

$25.98\ \text{cm}^2$

Q4a

4a

Triangle PSQ and SQR are such that $PS = SQ = QR$. Sides $PQ = 15$ cm and QR $= 2x$ cm. Angle $PSQ = 120°$.



(a) Calculate the exact value of x.

[3]

(b) Calculate the area of the triangle PQR. Leaving your answer in surd form.

[2]

a)

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{2x}{\sin 30} = \frac{15}{\sin 120}$

$2x = \frac{15}{\sin 120} \times \sin 30$

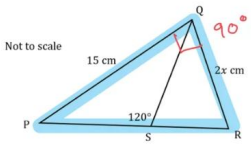
$2x = 5\sqrt{3}$

$x = \frac{5\sqrt{3}}{2}$

Q4b

4b

Triangle PSQ and SQR are such that $PS = SQ = QR$. Sides $PQ = 15$ cm and $QR = 2x$ cm. Angle $PSQ = 120°$.



(a) Calculate the exact value of x.

[3]

(b) Calculate the area of the triangle PQR. Leaving your answer in surd form.

[2]

b) $A = \frac{1}{2}bh$

$A = \frac{1}{2}(2x)(15)$

$x = \frac{5\sqrt{3}}{2}$

$= \frac{1}{2}(5\sqrt{3})(15)$

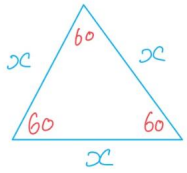
$A = \frac{75\sqrt{3}}{2}$

Q5

5

An artist is designing a triangular sculpture, made using three equal lengths of metal piping. When laid flat the sculpture covers 21.8 m^2.
Calculate the total length of metal piping needed. Giving your answer to the nearest cm.

[5]



$AREA = \frac{1}{2}ab\sin C$

$\frac{1}{2}x^2 \sin 60 = 21.8$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{4}x^2 = 21.8$

$\div \frac{\sqrt{3}}{4}$ $\div \frac{\sqrt{3}}{4}$

$x^2 = 50.344...$

$x = \sqrt{50.344...} = \pm 7.09...$

IGNORE NEGATIVE

$= 7.09..$ m

TOTAL LENGTH

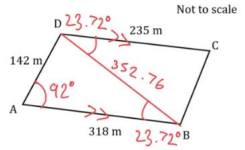
$7.09.. \times 3 = 21.286...$

21.29 m

Q6

6

Unicorns are kept in a field as shown in the diagram below. The angle between fence AB and AD is 92°. AB and CD are parallel.



To be happy unicorns need at least 2222 m² each. Calculate the maximum number of unicorns that can happily be kept in the field.

[9]

BD $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 142^2 + 318^2 - 2(142)(318)\cos 92$

$a = \sqrt{124439.8433} = 352.7603\ldots$

ABD $\frac{\sin B}{b} = \frac{\sin A}{a}$ $\frac{\sin B}{142} = \frac{\sin 92}{352.76}$

$\sin B = \frac{\sin 92}{352.76} \times 142$

$\sin^{-1}(0.4022\ldots) = 23.72\ldots$

ABD = BDC PARALLEL LINES

NO NEED TO FIND MISSING SIDE AS AREA NEEDS 2 SIDES AND ANGLE

AREA $= \frac{1}{2}ab\sin C$

$\frac{1}{2}(142)(318)\sin(92) + \frac{1}{2}(235)(352.76)\sin(23.72)$

$22564.24609 + 16673.70126$

$= 39237.94735\ m^2$

$\div 2222 \quad = 17.6588\ldots$

17 UNICORNS

Q7a

7a

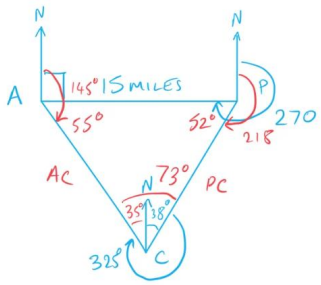
An emergency call is picked up by an ambulance and a police car about an accident. The police car is 15 miles due east of the ambulance and on a bearing of 038° from the accident. The ambulance is on a bearing of 325° from the accident.

(a) If both vehicles take the shortest distance to drive to the accident who will get there first? You must show all working.

[4]

(b) State one assumption you have made for your answer in part (a).

[1]



a) USING BACK BEARINGS

CFROMP $38 + 180 = 218$

APC $= 270 - 218 = 52°$

CFROMA $325 - 180 = 145$

CAP $= 145 - 90 = 55°$

SINE RULE

$\frac{15}{\sin 73°} = \frac{AC}{\sin 52} = \frac{PC}{\sin 55}$

$AC = \frac{15}{\sin 73} \times \sin 52 = 12.360\ldots$

$PC = \frac{15}{\sin 73} \times \sin 55 = 12.84\ldots$

AMBULANCE TRAVELS 12.4 MILES

POLICE CAR TRAVELS 12.8 MILES

AMBULANCE WILL ARRIVE FIRST

Q7b

7b

An emergency call is picked up by an ambulance and a police car about an accident. The police car is 15 miles due east of the ambulance and on a bearing of 038° from the accident. The ambulance is on a bearing of 325° from the accident.

(a) If both vehicles take the shortest distance to drive to the accident who will get there first? You must show all working.

[4]

(b) State one assumption you have made for your answer in part (a).

[1]

b)

BOTH VEHICLES TRAVEL AT SAME SPEED

OR

NEITHER IS CAUGHT IN TRAFFIC

OR

BOTH ROUTES ARE STRAIGHT DIRECT ROADS

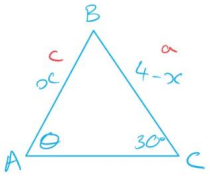
Q8

8

A triangle ABC has sides $AB = x$ cm, $BC = (4 - x)$ cm, angle $BAC = \theta$ and angle $BCA = 30°$.

Given that $\sin\theta = \frac{1}{\sqrt{2}}$, show that $x = 4(\sqrt{2} - 1)$.

[5]



$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\quad \frac{4-x}{\sin\theta} = \frac{x}{\sin 30}$

$\sin 30(4-x) = \sin\theta(x)$ $\quad \sin 30 = \frac{1}{2}$ $\quad \sin\theta = \frac{1}{\sqrt{2}}$

$\frac{1}{2}(4-x) = \frac{1}{\sqrt{2}}x$ $\quad \times 2$

$\times 2$

$4 - x = \sqrt{2}x$

$4 = \sqrt{2}x + x$ FACTORISE

$4 = x(\sqrt{2}+1)$

$x = \frac{4}{\sqrt{2}+1}$ RATIONALISE USING $(\sqrt{2}-1)$

$x = \frac{4(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{4(\sqrt{2}-1)}{2+\sqrt{2}-\sqrt{2}-1}$

$x = 4(\sqrt{2}-1)$

Q9

9

A triangle ABC has sides $AB = 3x$ cm, $AC = (x + 5)$ cm and angle $BAC = 150°$.
The area of the triangle is $7\frac{1}{4}$ cm^2.

Find the ratio of the angles of the triangle, to the nearest degree.
Leave your answer in simplest form.

[6]

AREA $= \frac{1}{2}ab\sin C$

$\frac{1}{2}(3x)(x+5)\sin 150 = 7\frac{1}{4}$

$\sin 150 = \frac{1}{2}$

$\frac{1}{4}(3x^2 + 15x) = \frac{29}{4}$

$\times 4$ $\times 4$

$3x^2 + 15x = 29$

$3x^2 + 15x - 29 = 0$

$x = \frac{-15 + \sqrt{573}}{6}$

IGNORE NEGATIVE SOLUTION

$b = x + 5 = \frac{15 + \sqrt{573}}{6} = 6.489$

$c = 3x = \frac{-15 + \sqrt{573}}{2} = 4.469$

COSINE TO FIND BC

$a^2 = (6.489)^2 + (4.469)^2 - 2(6.489)(4.469)\cos 150$

$a^2 = 112.307$

$a = \sqrt{112.307} = 10.597...$ $a = 10.6$

SINE RULE TO FIND ANGLES

$\frac{\sin 150}{10.6} = \frac{\sin B}{6.489} = \frac{\sin C}{4.469}$

$\sin B = \frac{\sin 150 \times 6.489}{10.6} = 0.306$

$B = \sin^{-1}(0.306) = 17.823... = 18$

$A = 150$ $B = 18$ $C = 12$

$150 : 18 : 12$

$25 : 3 : 2$